Correction to attenuation treatment in the Monterey-Miami Parabolic Equation Model

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The purpose of this paper is to notify those in the community who have used, or are using, the Monterey-Miami Parabolic Equation (MMPE) Model of an error in previous implementations that affects the computation of volume loss. The error has been corrected and an updated version is now available. Thorough analysis of the results are provided here, including comparison with exact modal attenuation factors for bottom loss parameters, which indicate the model is now performing properly.

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I. INTRODUCTION

In the early 90's, a numerical code known as the University of Miami Parabolic Equation (UMPE) Model was documented and made available to the general research community.[1] This model was based on the split-step Fourier (SSF) technique[2], and had been adapted from previous versions developed by Fred Tappert at the University of Miami. A subsequent version, known as the Monterey-Miami Parabolic Equation (MMPE) Model, was developed in the mid-90's that was more streamlined and user-friendly. This code was thoroughly tested against several existing benchmark scenarios and was found to perform reasonably well during the Shallow Water Acoustic Modeling Workshop help in Monterey, CA in 1999 (SWAM'99).[3] Since that time, however, various researchers who have used the MMPE Model have noticed issues with bottom boundary interactions. It was assumed that this was due to the manner in which the SSF technique treated the bottom interface by introducing mixing functions that smeared the boundary discontinuity over some finite depth extent. While this may certainly cause some deficiencies, some researchers (including the authors) noted specific problems with the treatment of bottom attenuation.

As detailed in Ref. [3], the treatment of attenuation is the SSF algorithm is accomplished simply by defining a damping vector in depth at each range step, given by

$$e^{-ik_0\Delta r U_{loss}(z)} = e^{-\Delta r \alpha(z)} \ ,$$

where U_{loss} becomes part of the z-space, "potential energy" propagator function. This simplistic approach has been questioned in the past with respect to proper treatment of individual modal attenuation. Until now, this approach has never formally been validated as accurate.

Bottom attenuation presents subtle issues in shallow water propagation. It is known to be notoriously difficult to invert for values with any degree of accuracy.[e.g., 4] And for most shallow water sediments, it typically has a relatively small value. This makes it particularly difficult to assess when energy is trapped in the waveguide beyond the critical angle of incidence. Still, for many bottom interactions, it would be expected to become a noticeable effect over long range.

Careful scrutiny of the code recently revealed the error, in which the attenuation factor was being multiplied by the depth mesh rather than the range step. In the code, this was simply a misplaced “dz” where it should have been “dr”. With nearly 2000 lines of code, it was an easy error to miss. The corrected code should provide users who have noted previous problems with the results they were expecting.

In the following sections, we show results from the MMPE model, both with the error and with a corrected version, for a simple Pekeris waveguide. The lack of sensitivity to bottom attenuation is illustrated for point source calculations. Correct evaluation of bottom loss effects requires the examination of the propagation of individual modes. Exact values for mode attenuation are computed and compared with the results of the corrected code. We conclude the paper with a summary of findings.

II. POINT SOURCE RESULTS

For all of the work presented here, we assume the environment is that of a Pekeris waveguide of depth 150 m. The sound speed in the water column is 1500 m/s with a density of 1.0 g/cc, while the bottom has a sound speed of 1600 m/s with a density of 1.2 g/cc. The bottom attenuation is provided in units of dB/km/Hz, and the water column is assumed to be lossless.

Figure 1 illustrates the difficulty in observing the error for typical values of attenuation when a point source is utilized. The point source is in the middle of the water column at 75 m transmitting a CW tone at 500 Hz. Specifically, it compares the transmission loss (TL) trace at 75 m from the old code (with the error present) with the new code (corrected) when the attenuation parameter is set to 0.1 dB/km/Hz, a typical value for a sediment with the associated sound speed and density defined above. The two plots are nearly identical with differences never exceeding 1 or 2 dB out to 10 km range.

A more obvious difference is observed if the attenuation factor is increased to a relatively large value (perhaps unrealistic for this bottom type) of 0.5 dB/km/Hz. A comparison of these results is presented in Fig. 2. The differences between the two curves now approach 5 dB at 10 km range. Still, the curves are remarkably similar, and it would be difficult to associate these differences with an implementation error vice some other computational parameter tuning (e.g., range step or depth mesh sizes). Therefore, a more detailed analysis is desired, and we now turn our attention to results for propagation of individual modes.

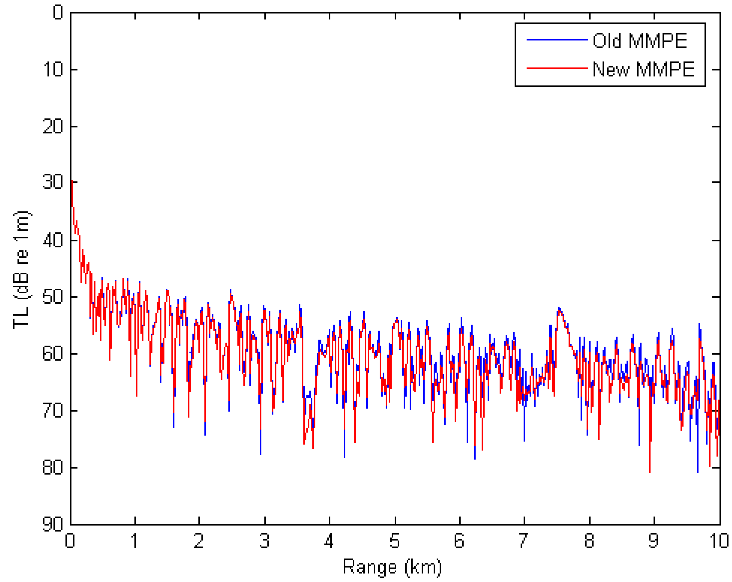

Figure 1. Transmission loss trace at 75 m depth in Pekeris waveguide described in text for point source at 75 m transmitting 500 Hz tone. Results presented for old code (blue curve) and updated code (red curve) with bottom attenuation factor set to dB/km/Hz.

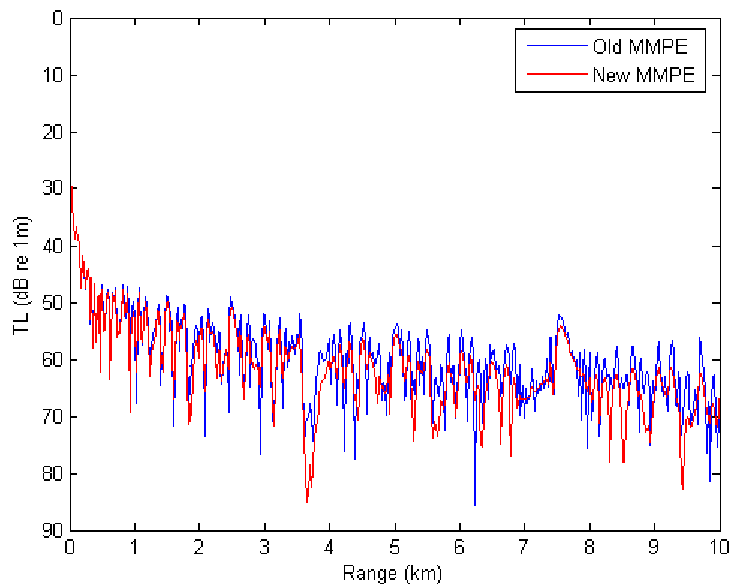

Figure 2. Same results as Figure 1 with bottom attenuation factor increased to dB/km/Hz.

III MODE PROPAGATION RESULTS

A simple mode code was developed to obtain the eigenvalues for the trapped modes of this waveguide. These values were then fed into an altered version of the MMPE Model that used an analytic description of the mode structure to define the starting field. Specifically, the mode shapes were sinusoidal in the water column and decayed exponentially in the bottom. Differences in the mode eigenvalues and eigenfunctions when attenuation was non-zero were assumed to be small and neglected. In what follows, we shall examine results for modes 5, 15, and 25. The corresponding horizontal wavenumber components were computed to be 2.09182731 m^{-1}, 2.07118063 m^{-1}, and 2.02933430 m^{-1}, respectively.

Figure 3 shows the TL trace at 75 m for modes 5, 15, and 25 when attenuation was set to $\alpha = 0$ dB/km/Hz. Both old and new versions of the code produce the same results in this case. As expected, all three curves correspond simply to cylindrical spreading, i.e. $TL(\text{re}1\text{m}) = 10\log r$. Some small fluctuations (≤ 0.5 dB) in the TL trace are observable for mode 25 at long range. This is presumably due to the use of the standard normal mode eigenfunctions with the Thompson-Chapman wide-angle PE operators,[5,6] and possibly the use of mixing functions to smear the bottom boundary discontinuity.[1,3] This effect is not considered significant in this analysis.

In order to create noticeable effects of bottom attenuation, the bottom loss parameter is now set to $\alpha = 0.5$ dB/km/Hz. In Fig. 4, similar TL traces for the modes are displayed when the old code containing the error is used. We find that there is little effect on the mode amplitudes, although a larger effect should be expected.

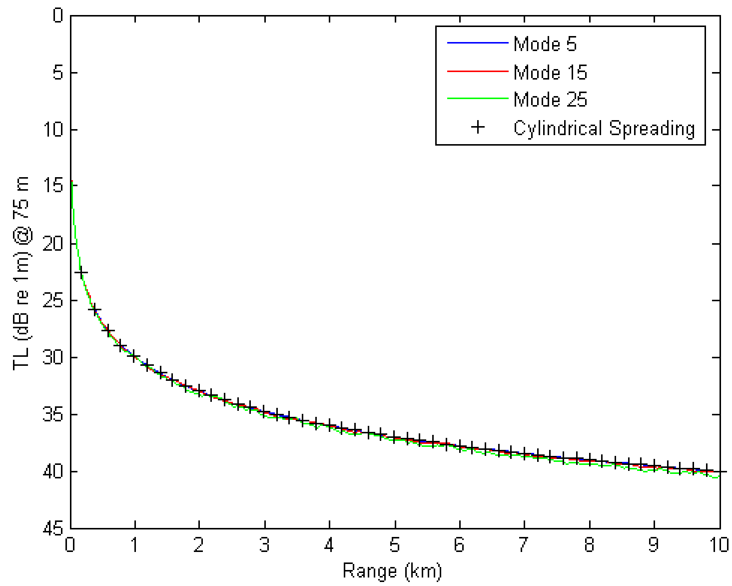

Figure 3. Transmission loss trace at 75 m depth in Pekeris waveguide described in text for modes 5 (blue curve), 15 (red curve), and 25 (green curve) at 500 Hz with no attenuation. Ideal cylindrical spreading plotted as black crosses (+).

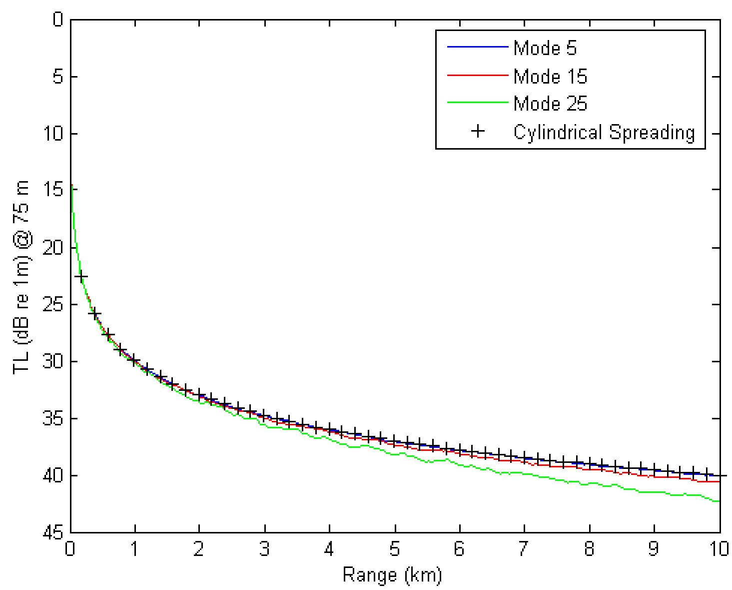

Figure 4. Same as Figure 3 but modal transmission loss results computed using old code with bottom attenuation factor set to dB/km/Hz.

To determine what the effect should be, we must compute the specific modal attenuation coefficients. A perturbational treatment of the loss similar to that presented by Jensen, et al.[7] provides a means of computing the mode attenuation coefficients analytically. Specifically, we define each mode attenuation coefficient as

$$\delta_{m,e} = \frac{\omega}{K_m} \int_0^{\infty} \frac{\alpha_e(z)}{c(z)} \Psi_m^2(z)\, dz \,,$$

where $\omega = 2\pi f$ is the angular frequency, K_m is the horizontal component of the mode wavenumber, $c(z)$ is the depth-dependent sound speed profile, $\Psi_m(z)$ is the mode eigenfunction, and $\alpha_e(z)$ is the depth-dependent exponential attenuation factor. Note that the units of α_e are m^{-1}, such that a conversion to dB units is accomplished by defining

$$\delta_m(\mathrm{dB/km/Hz}) = \delta_{m,e}\left(\mathrm{m}^{-1}\right) \times 8.686 / f\,(kHz) \;.$$

Furthermore, the mode eigenfunctions are normalized according to

$$\int_0^{\infty} \frac{\Psi_m^2(z)}{\rho(z)} dz = 1 \;.$$

The results of this calculation provide mode attenuation coefficients of 5.1509x10^{-5} m^{-1}, 4.9991x10^{-4} m^{-1}, and 1.7293x10^{-3} m^{-1} for modes 5, 15, and 25, respectively.

Figure 5 displays the TL traces for the three modes along with the exact predictions defined by $TL(\mathrm{re\,1m}) = 10\log r + \delta_m r$. The trace for cylindrical spreading only is also included for reference. We note that the lowest modes (~5 or lower) still show little effect from this relatively large bottom attenuation. However, the higher modes begin to show significant additional loss at longer ranges. Furthermore, the

MMPE predictions of the mode propagation loss agree extremely well with theoretical predictions. Thus, the code has been corrected and is now working properly.

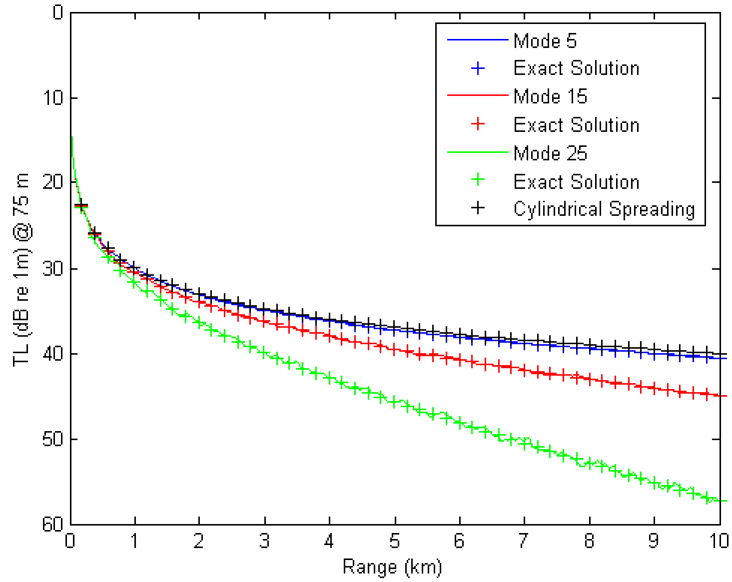

Figure 5. Similar to Figure 4 but modal transmission loss plots now computed using updated code with bottom attenuation factor set to dB/km/Hz. Colored crosses (blue, red, and green) also included to show analytical predictions of mode attenuation (for modes 5, 15, and 25, respectively).

IV SUMMARY

In this paper, an error in a previous implementation of the MMPE Model was identified and illustrated. This error caused the code to underestimate the effects of volume attenuation in the environment. This was most clearly observable for bottom loss calculations. The code has been corrected, and was tested against analytical predictions of individual mode attenuation factors. The results agree quite nicely, indicating that the corrected code now properly treats attenuation effects. Updated versions of the code may now be obtained from the Ocean Acoustics Library website at http://www.hlsresearch.com/oalib/PE/MMPE.

ACKNOWLEDGEMENTS

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