A SHORT NOTE ON THE 2D-MC PROGRAM

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Introduction

The 2D-MC program computes numerically the time dependent solution of Euler and continuity equations with adiabatic condition in a waveguide consisting of a homogeneous fluid layer overlying a rigid bottom, using the method of Characteristics (MC) with Semi-Lagrange scheme[1], which is not constrained by Courant-Friedrichs- Lewy (CFL) condition. Comments and Bug report are welcome.

Mathematical Formulation and Numerical Scheme

The fundamental equations are written in cylindrical coordinates. The pressure release surface and rigid bottom are assumed at the upper and lower boundaries, respectively.

One dimensional Euler and continuity equations with adiabatic condition are written in the form:

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} \qquad (1)$$

$$\frac{\partial p}{\partial t} = -\rho c^2 \frac{\partial u}{\partial x} \qquad (2)\ .$$

Multiplying Eq. (1) by $\pm c^2$ and adding to Eq. (2), we obtain

$$\frac{\partial f^+}{\partial t} + c\frac{\partial f^+}{\partial x} = 0 \qquad (3)$$

$$\frac{\partial f^-}{\partial t} - c\frac{\partial f^-}{\partial x} = 0 \qquad (4)$$

where

$$f^+ = \rho c u + p, \qquad (5)$$

$$f^- = \rho c u - p \qquad (6)\ .$$

Equations (3) and (4) are advection equations with sound speeds of + c and $-c$, respectively. Quantities f^+ and f^- are advected along its characteristics. Therefore new values at the next time are determined if one can find the up-wind values along the characteristics as shown in figure 1.

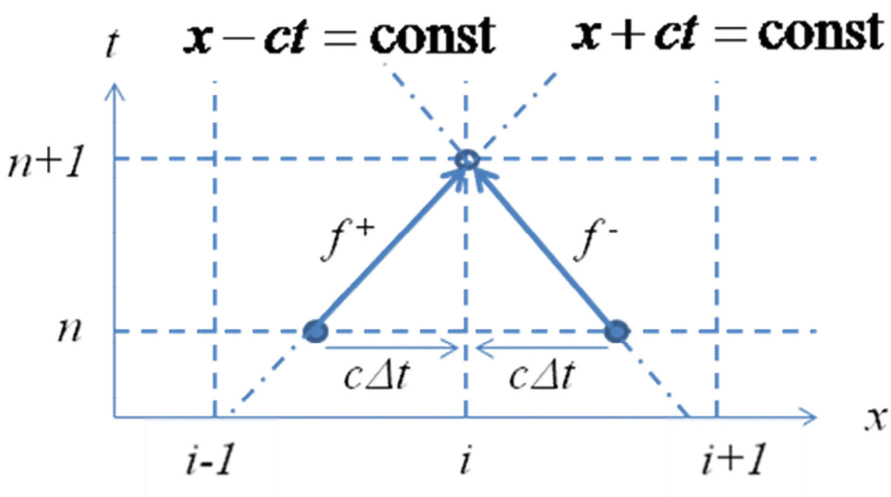

Figure1 The up-wind scheme of advection along the characteristics

When CFL=1, f^+ and f^- propagate the values from a certain cell to the adjacent cell during time iteration. When CFL is not a natural number, Constrained Interpolation Profile (CIP) method[2-3] is applied. By adding and subtracting Eqs (5) and (6), the pressure and particle velocity are therefore

$$p=\frac{f^+-f^-}{2} \tag{7}$$

$$u=\frac{f^++f^-}{2\rho c} \tag{8}$$.

For two dimensional cases, the fundamental equations cannot be exactly written in the form of advection equations. However, the equations can be solved using a directional splitting method. First one solves the advection equation in range direction, and then solves the advection equation in depth direction. An additional non-advection term is solved using a finite difference/element method after advection.

Mesh Grid

This program uses square grids and all physical quantities (the pressure and particle velocity) are collocated.

The 2D-MC Program

The 2D-MC program is consisted of several modules written in Matlab. These modules are:

MC.m: Main program, which is the main routine of 2-D MC and contains input parameters, mesh size, etc.

cip_2d.m: Subroutine to compute advection.

nmode_rigid.m: Subroutine to compute the normal mode solution.

Input Parameters

Input parameters are given in the main program. Variables are:

f: frequency in Hz

c_0: sound speed in water column

H: water depth

zs: source depth

zr: receiver depth

tn: number of time iteration

flag: when flag = 0, the program computes propagation loss of CW at t = tnΔt and compares with normal mode solution. When flag is a natural number, the program computes PCW propagation and runs an animated image of the pressure fields. The number denotes the pulse length (e.g. flag =2 generates 2-wavelength PCWs). However, the pulse is modulated by hanning window.

Mesh size of space is one twentieth of the wavelength as a default. The time step is automatically determined by satisfying CFL=1.

Comments on Computational Time and Accuracy

No difference would be found in computational time between the Finite Difference Time Domain (FDTD) and MC methods per iteration, the FDTD is, however, constrained by more severe CFL condition:

$$c_0 \Delta t \sqrt{(1/\Delta r)^2 + (1/\Delta z)^2} \leq 1 \qquad (9) .$$

When the square grid is applied, the maximum value is CFL=$1/\sqrt{2}$ so the FDTD requires at least 1.4 times computational time.

Furthermore, the phase properties of the MC are more accurate than the FDTD. The difference is not prominent in low frequencies but would be found as frequencies increase.

References

1. P.K.Smolarkiewicz and J.A.Pudykiewicz, “A Class of Semi-Lagrange Approximations for Fluids,” J. Atmospheric Science, 49 (1992) pp.2082-2096.
2. T.Yabe, F.Xiao, and T.Utsumi, ”Constrained Interpolation Profile Method for Multiphase Analysis,” J. Comput. Phys., 169(2001) pp.556-593.
3. Takashi Yabe, Takayuki Utsumi, and Yohichi Ogata, *CIP Method* (in Japanese), Morikita Publishing, 2003.